# Univariate Analysis
# Seasonality

Applied Econometrics

# Seasonality

- Most of the data base offers its data for frequencies different to year. Then, we can employ quarterly or monthly data.
- This has the apparent advantage of increasing the sample. But this is not exactly true.
- Even worse, the use of this data implies the appearance of a new component: Seasonality.

# Seasonality

- It is not always so easy to handle.
- Some people prefer to remove it by filtering the data.
- However, this implies that we are not working with raw data. Rather, we use transformed data with may not be can inappropriate version of the original ones.
- Thus, if we leave this component in the variable, it should be modelled as the rest.

# Types of Seasonal Models

Deterministic model:

$$y_t = \sum_{i=1}^{s} \gamma_i D_i + \mu + \beta t + u_t$$

Where Di is a variable which takes the value 1 for the s-th seasonal period and 0 otherwise.

If we have quarterly data then s=4, (m=12 for monthly data and so on)

# Types of Seasonal Models

- There are many possibilities of considering stochastic seasonality.
- The most popular is the use of the Seasonal ARIMA model, commonly referred to as $ARIMA_s(P,D,Q)$

$$
\begin{aligned}
(1-\Phi_1 L^s - .. - \Phi_P L^{Ps})(1-L^s)^D y_t \\
= \delta + \left(1+\Theta_1 L^s - .. - \Theta_Q L^{Qs}\right) u_t
\end{aligned}
$$

# Types of Seasonal Models

ARIMA$_4$(1,0,0)

$$(1-\Phi_1 L^4)y_t = \delta + u_t$$

Or, equivalently,

$$y_t = \delta + \Phi_1 y_{t-4} + u_t$$

# Types of Seasonal Models

$ARIMA_{12}(0,0,1)$

$$y_t = \delta + (1 + \Theta_1 L^{12})u_t$$

Or, equivalently,

$$y_t = \delta + u_t + \Theta_1 u_{t-12}$$

# Types of Seasonal Models

- Previous models are purely seasonal.
- But, the variables may exhibit this component besides cycle and trend.
- We can use a mixed model to that end, getting a multiplicative model ARIMA(p,d,q,) x $\text{ARIMA}_s$(P,D,Q)

$$
\begin{aligned}
(1-\Phi_1 L^s - .. - \Phi_P L^{Ps})(1-L^s)^D y_t \\
= \delta + \left(1+\Theta_1 L^s - .. - \Theta_Q L^{Qs}\right) u_t
\end{aligned}
$$

# Types of Seasonal Models

These models can be stated as follows:

$$\left(1-\Phi_1 L-\cdots-\Phi_p L^p\right)(1-\Phi_1 L^s-\cdots-\Phi_P L^{Ps})(1-L)^d(1-L^s)^D y_t = \delta + \left(1+\Theta_1 L^s-\cdots-\Theta_Q L^{Qs}\right)\left(1+\Theta_1 L^s-\cdots-\Theta_Q L^{Qs}\right)u_t$$

# Types of Seasonal Models

Airline models

$$(1-L)(1-L^s)y_t = \delta + (1+\theta_1 L)(1+\Theta_1 L^s)u_t$$

Agustin Maravall advice

$$(1-L)^2(1-L^s)^2 y_t = \delta + (1+\theta_1 L+\theta_2 L^2)(1+\Theta_1 L^s+\Theta_2 L^{2s})u_t$$

# Testing for seasonal unit root

- Trends can also appear in seasonal variables.
- They can be deterministic or stochastic.
- The existence of unit roots is not so direct, given that they can appear in different frequencies.
- Thus, it is no easy handle with seasonal unit roots, once again.

# Deterministic trends

This case is similar to the one analyzed without taking into account the presence of seasonal component

$$y_t = \alpha + \beta t + u_t$$

where $u_t$ follows a type of mixed ARIMA process.

# Stochastic trends

- This case should be associated to the presence of unit roots.
- These unit roots may appear in different frequencies.
- The most general case can be defined as follows:

$$(1 - L^s)y_t = \delta + \eta(L)u_t$$

where the $\eta(L)$ polynomial has all its roots out of the unit circle

# Stochastic trends

- Let us consider the case where s =4 (quarterly data)
- Then, we should take into account that:

$$\left(1-L^4\right)=\left(1-L^2\right)\left(1+L^2\right)$$
$$=(1-L)(1+L)\left(1+L^2\right)$$

- Consequently, we have four unit roots, each of the associated to a different frequency.

# Stochastic trends

- The non-seasonal unit root for the long-run frequency is captured by $(1 - L)$.
- The seasonal unit roots are captured by $(1 + L)$ and $(1 + L^2)$
- $(1 + L)$ represents the case of a double seasonal peak during a year (spring/fall, for instance)
- $(1 + L^2)$represents the case of a single seasonal peak during a year (turrón

# HEGY test

- Hylleberg, Engle, Granger and Yoo (1990) proposes a method for testing for unit roots in seasonal variables.
- It takes into account the decomposition of the $(1\text{-}L^4)$ polynomial.
- It is commonly referred to as the HEGY test

# HEGY test

$$y_{4t} = \mu + \beta t + \sum_{i=1}^{s-1} \gamma_i D_{it} + \pi_1 y_{1t-1} + \pi_2 y_{2t-1}$$

$$+ \pi_3 y_{3t-2} + \pi_4 y_{3t-1} + \sum_{i=1}^{m} \phi_i \, y_{4t-i} + u_t$$

$$y_{1t} = (1 + L)(1 + L^2) y_t$$
$$y_{2t} = -(1 - L)(1 + L^2) y_t$$

# HEGY test

- If $\pi_1$=0, then we have an unit root in the non-seasonal frequency.
- If $\pi_2$=0, then we have an unit root at the $\pi$ seasonal frequency.
- If $\pi_3$=$\pi_4$=0, then we have unit roots at the $\pi/2$ seasonal frequency.

# HEGY test

$$y_{4t} = \pi_1 y_{1t-1} + \pi_2 y_{2t-1} + \pi_3 y_{3t-2} + \pi_4 y_{3t-1} + \sum_{i=1}^{m} \phi_i \, y_{4t-i} + u_t$$

If $\pi_2=\pi_3=\pi_4=0$, then we have unit roots in all the seasonal frequencies.

If $\pi_1=\pi_2=\pi_3=\pi_4=0$, then we have unit roots in all the frequencies. This implies that the variable is seasonal integrated and first